

Closing Tue: Sup. 6-7

Closing Wed: Sup. 8-9

## Supp. 7 and 8: Analysis of Cost

### Recall

### Total Amounts

$$TR(q) = (\text{price})(\text{quantity})$$

$$TC(q) = FC + VC(q)$$

-  $FC$  = fixed costs

-  $VC(q)$  = "costs to make  $q$  items"

$$P(q) = TR(q) - TC(q) = \text{Profit}$$

### Marginals (one-item increments)

$$MR(q) = TR(q+1) - TR(q)$$

= revenue for selling next item

$$MC(q) = TC(q+1) - TC(q)$$

= cost for making next item

**Entry Task:** Get out Blinkos graph.

1. Draw a *tangent* line at  $q = 24$  hundred Blinkos.
  - Estimate the slope of this line.
  - What does this represent?

2. Find a quantity at which the slope of the *tangent* line is equal 2.75 dollars/item.

1] DRAW LINE (SEE NEXT PAGE)

TWO PTS: (0, 15) (38, 50)

$$\text{SLOPE} = \frac{50 - 15}{38 - 0} = \frac{35}{38} \approx 0.9210$$

So  $MC(24) = 0.92$  dollars/item

⇒ THE 2401<sup>st</sup> BLINKO COSTS \$0.92 TO PRODUCE.

2] DRAW REFERENCE LINE (SEE NEXT PAGE).  
SLIDE PARALLEL UNTIL IT LOOKS LIKE A TANGENT.

$q \approx 31$  HUNDRED BLINKOS

THUS,  $MC(31) \approx 2.75$  \$/item  
IT COSTS ABOUT \$2.75 TO PRODUCE THE 3101<sup>st</sup> BLINKO.

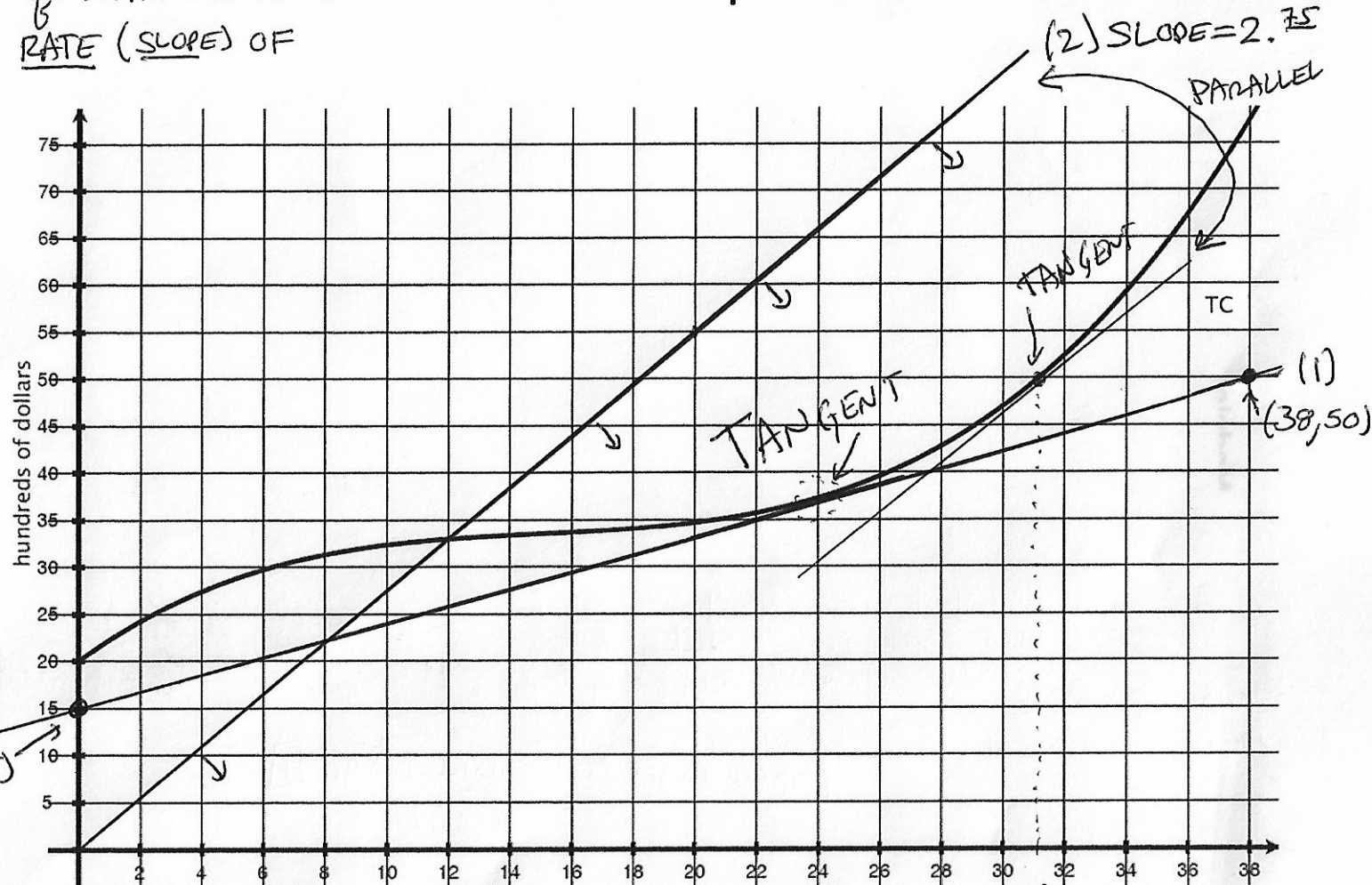
ASIDE:  $FC = 20$  HUNDRED DOLLARS (\$2000)

Units are in hundreds,  
 1 Blinko:  $q = 0.01$  hundred Blinko

(1) MARGINAL COST AT  
 IS THE SAME AS THE  
 THE SECANT FROM  
 2400 ITEMS  $\Leftrightarrow q=24$   
 TO  
 2401 ITEMS  $\Leftrightarrow q=24.01$   
 WHICH LOOKS LIKE A  
 TANGENT LINE

$q = 24$  HUNDRED ITEMS  
 RATE (SLOPE) OF

2 PTS  $(0, 15), (38, 50)$   
 $SLOPE = \frac{50 - 15}{38 - 0} = 0.9210$



(2) GIVE SLOPE = 2.75  
 DRAW REFERENCE LINE!  $(0, 0), (10, 27.5), (20, 55), \dots$   
 SLIDE RULER PARALLEL UNTIL IT LOOKS LIKE A TANGENT.

(2) SLOPE = 2.75  
 PARALLEL

AROUND 31

Now suppose we sell Blinkos at a price of \$2.00 per Blinko.

1. Draw the TR graph
2. How many quantities should you sell in order to maximize profit?

$$TR(q) = (\text{PRICE}) (\text{QUANTITY}) = 2q$$

DRAW A REFERENCE LINE WITH SLOPE 2,  $(0,0)$   $(10,20)$   $(30,60)$  (see next page)

PROFIT = "VERTICAL GAP THAT TR IS ABOVE TC"

MAX PROFIT = "LARGEST VERTICAL GAP WHEN TR IS ABOVE TC"

COOL OBSERVATION MAX PROFIT OCCURS AT A QUANTITY WHERE THE TANGENT SLOPES MATCH ( $MR = MC$ ).

(SEE NEXT PAGE FOR VISUALS)

WE FIND

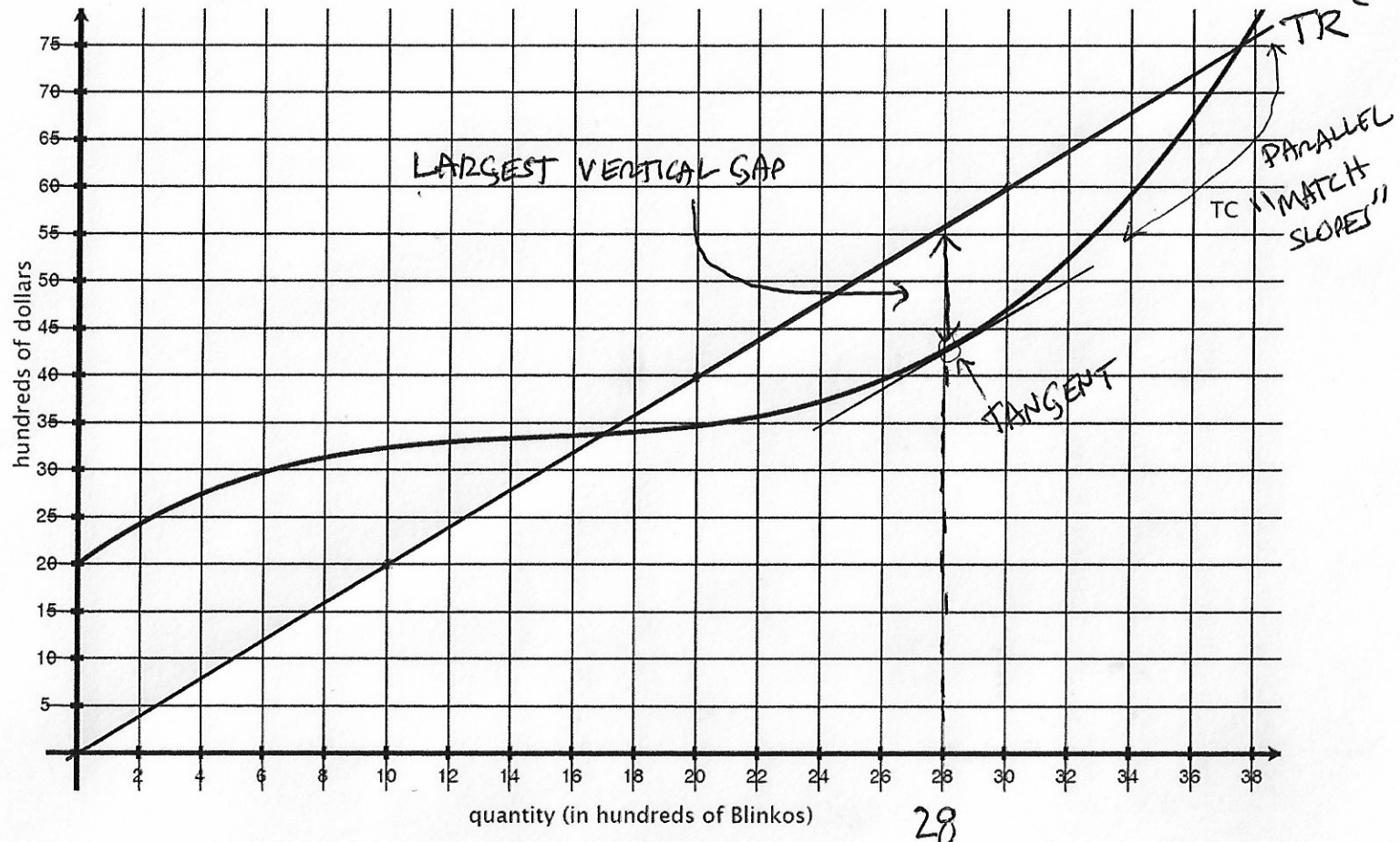
$$q \approx 28$$

AND

MAX PROFIT  $\approx$  13 HUNDRED DOLLARS

\$1300

WHEN PRICE = \$2/item, TR is a line with slope = 2



MAX PROFIT OCCURS AROUND  $q = 28$  HUNDRED BLINKOS.

AND

MAX PROFIT = "VERTICAL GAP AT 28"  $\approx$   $\underbrace{56}_{TR(28)} - \underbrace{43}_{TC(28)} = 13$  HUNDRED DOLLARS

MAX PROFIT = \$1300



# Max Profit Methods Summarized

## Method 1: Largest Gap

Find the largest vertical gap when TR is above TC.

Pro = quick

Con = hard to be accurate

## Method 2: Match Slopes

Find where TR and TC have the same tangent slopes. Best way if given TR and TC graphs!

## Method 3: $MR = MC$

Find where it switches from  $MR > MC$  to  $MR < MC$

*Note:*  $MR > MC \rightarrow$  profit increasing

$MR < MC \rightarrow$  profit decreasing

Pro = precise (algebra)

Con = need MR/MC formulas

## Sup. 6-7 HW / Problem 2 Hints:

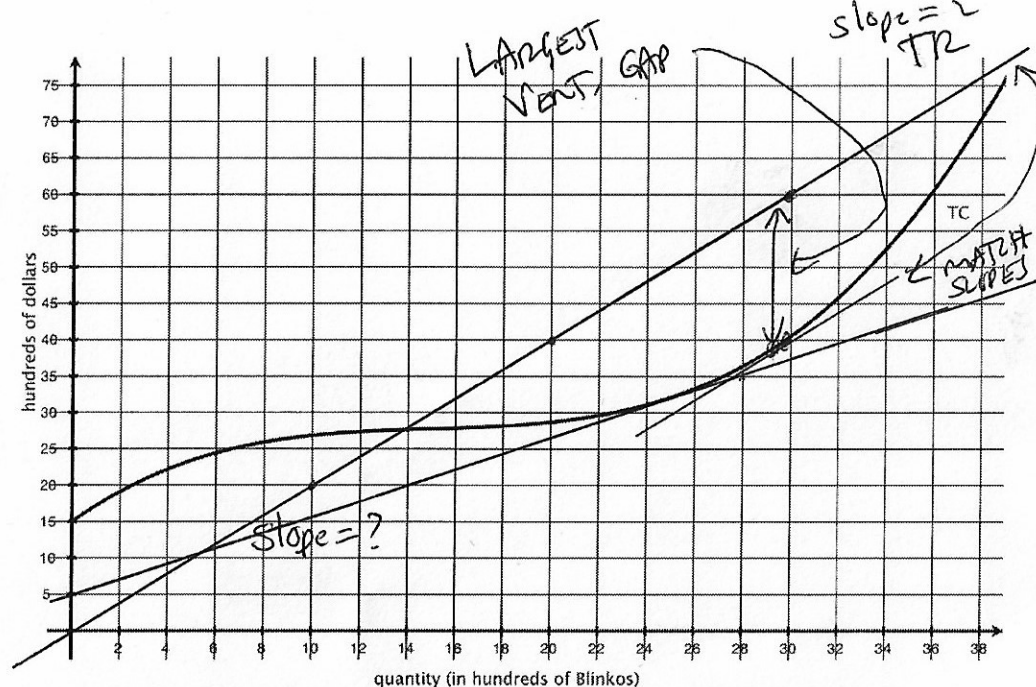
Do you know how to answer these...

(a) What is  $TC(38)$  and  $VC(38)$ ?

$$TC(38) = 70 \text{ hundred dollars}$$

SINCE  $FC = 15$  hundred dollars

$$VC(38) = 70 - 15 = 55 \text{ hundred dollars}$$



(b) How would you estimate MC at 24 hundred items?

$\approx$  SLOPE OF TANGENT AT  $q=24$

GET TWO PTS!

$$(14, 20) \quad (28, 35)$$

$$\frac{35 - 20}{28 - 14} = \frac{15}{14} \approx 1.07/\text{item}$$

The 2,401<sup>st</sup> item will cost about \$1.07 to produce

(c) If the items sell for \$2 per item what is the maximum profit?

DRAW TR!

LARGEST VERTICAL GAP (MATCH-SLOPES!)

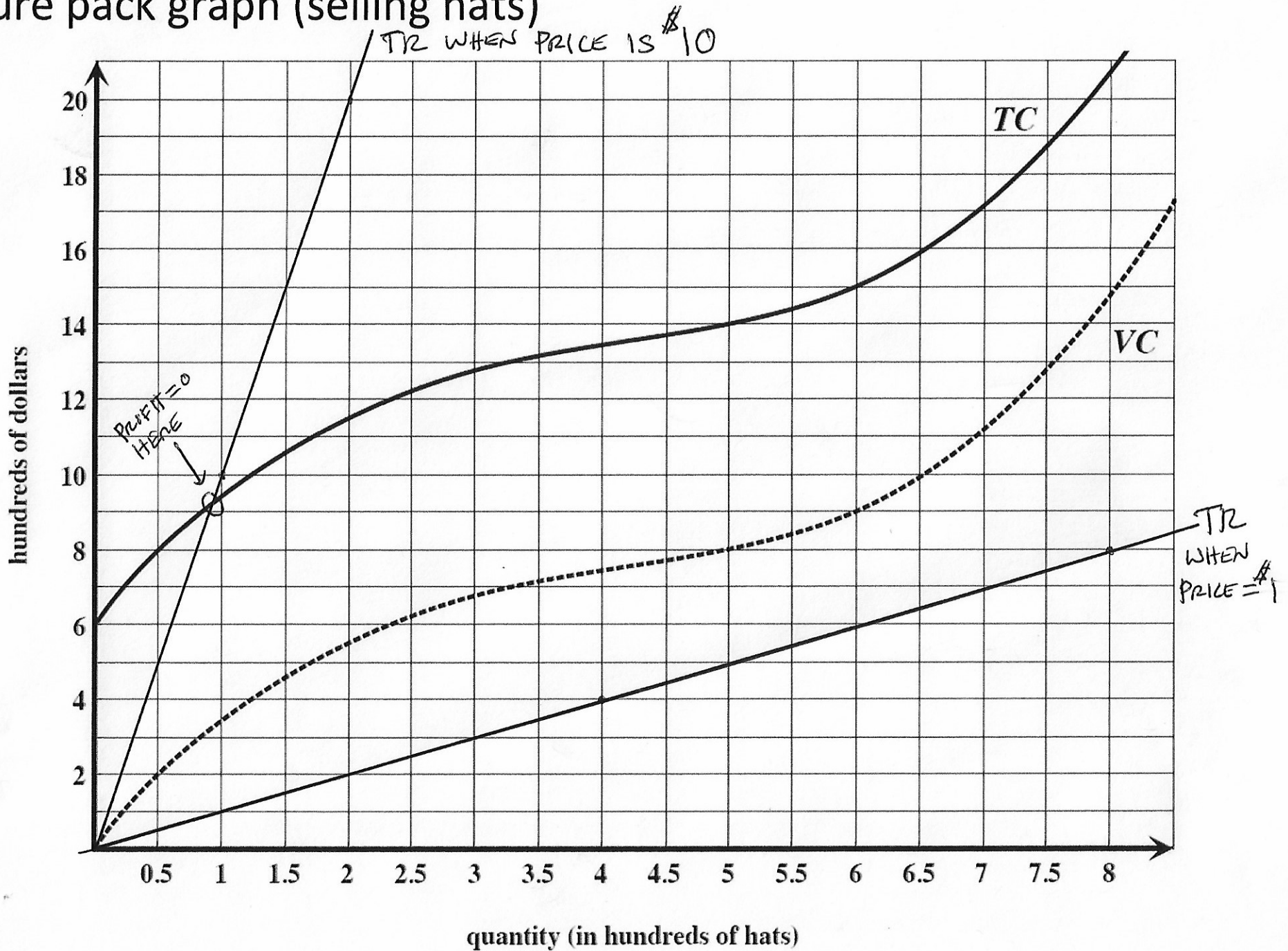
• OCCURS AROUND  $q=29$  or 30

$$\text{GAP} \approx 60 - 40 \approx 20 \text{ hundred dollars}$$

$$\approx \$2,000$$

# Sup. 8 - Analysis of Cost

Lecture pack graph (selling hats)



## Quick check of understanding:

- What is  $TC(0)$ ?

$$TC(0) = 6 \text{ HUNDRED DOLLARS}$$

$$FC = \$600 \text{ (FIXED COSTS)}$$

- What is  $TC(0.5)$ ?

$$TC(0.5) = 8 \text{ HUNDRED DOLLARS}$$

IF YOU PRODUCE 50 HATS IT WILL COST \$800 IN TOTAL (INCLUDING FC).

NOTE:  $VC(0.5) = 2$  HUNDRED DOLLARS

$$\text{AND } TC(0.5) - VC(0.5) = 6 = FC$$

ALWAYS!

## Analysis of Various Selling Prices:

Remember, if price is a constant, then TR is a diagonal line. Let's consider various prices (slopes).

- If price = \$1.00/item, draw TR.

Can you break even?

$(0,0), (5,5), \dots, (8,8)$  DRAW LINE (SEE GRAPH)

CAN'T BREAK EVEN. NO MATTER HOW MANY YOU SELL YOU WILL LOSE MONEY!  
IN FACT, YOU WILL ALWAYS LOSE MORE THAN FC!

- If price = \$10.00/item, draw TR.

Can you break even? (SEE GRAPH)

$(0,0), \dots, (1,10), \dots, (2,20), \dots$  DRAW LINE

IF YOU SELL MORE THAN 900 HATS THEN PROFIT WILL BE POSITIVE.

POSITIVE PROFIT IS POSSIBLE  
IF PRICE = \$10/HAT.



## Benchmark 1:

**Break Even Price (BEP)** is the **slope** of the lowest diagonal line that touches TC.

- If market price is smaller than BEP, then NO quantity will yield a positive profit.
- If market price is bigger than BEP, then it is possible to get a positive profit for some quantities.

## Example

Compute BEP for the hats graph.

- DRAW A TR LINE (DIAGONAL LINE) THAT JUST TOUCHES (TANGENT TO) TC.
  - FIND THE SLOPE (PRICE).
- TWO POINTS: (0,0) (4.5, 11)
- SEE GRAPH

$$\text{SLOPE} = \frac{11-0}{4.5-0} = 2.44 \text{ \$/HAT}$$

- IF price  $>$  \$2.44/HAT, IT IS POSSIBLE TO BREAK EVEN & MAKE A POSITIVE PROFIT.
- IF price  $<$  \$2.44/HAT, THEN POSITIVE PROFIT IS NOT POSSIBLE.

$$\text{BEP} = \$2.44$$

NOTE: THIS IS NOT A DESIRABLE PRICE FOR THE BUSINESS!  
WE HOPE THAT THE SELLING PRICE IS MUCH, MUCH HIGHER THAN BEP. BUT THIS IS A USEFUL BENCHMARK.

## Benchmark 2:

**Shutdown Price (SDP)** is the slope of the lowest diagonal line that touches VC.

- If market price is smaller than SDP, then NO quantity will allow you to recover any fixed costs (Shut down!)
- If market price  $p$  is bigger than SDP, then it is possible to recover at least some fixed costs for some quantities.

SO FOR OUR HAT BUSINESS  
IF HAT PRICES DROP BELOW \$1.50, SHUT DOWN.  
IF HAT PRICES ARE BETWEEN \$1.50 AND \$2.44,  
BUT, ULTIMATELY, WE NEED PRICES WELL ABOVE \$2.44/HAT

## Example

Compute SDP for the hats graph.

DRAW A TR LINE (DIAGONAL LINE) THAT JUST TOUCHES (TANGENT TO) VC.

FIND THE SLOPE (PRICE)

TWO POINTS: (0,0), (8,12)

$$\text{SLOPE} = \frac{12-0}{8-0} = \$1.50/\text{HAT}$$

SEE GRAPH

IF PRICE  $>$  \$1.50/HAT, IT IS POSSIBLE TO RECOVER SOME FIXED COSTS.

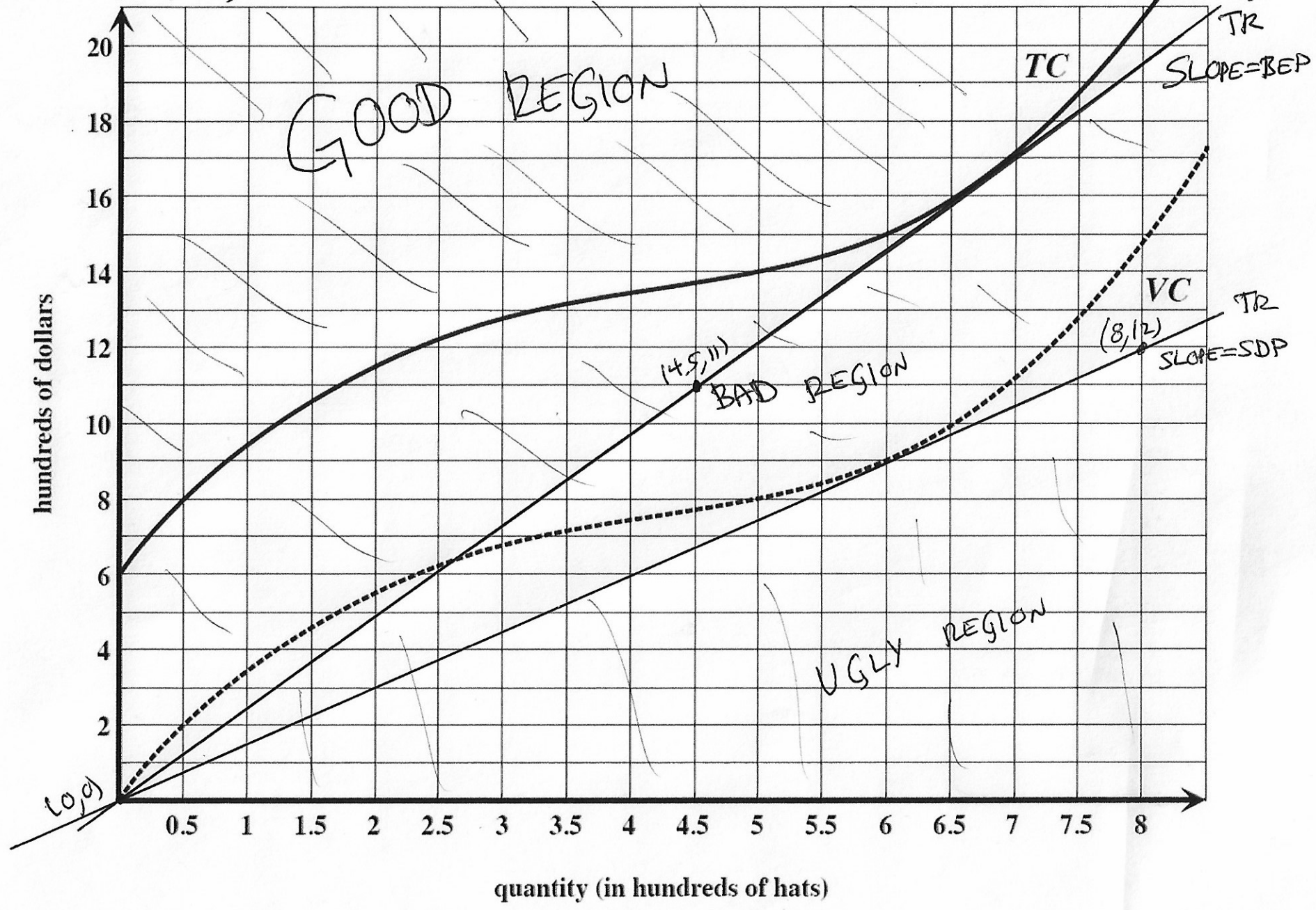
IF PRICE  $<$  \$1.50/HAT, THEN WE WILL LOSE MORE THAN FIXED COSTS IF WE PRODUCE & SELL ANY HATS (SHUT DOWN!)

$$\boxed{\text{SDP} = \$1.50}$$

STAY OPEN IN HOPES TO RECOVER SOME FIXED COSTS.

ABOVE \$2.44/HAT TO MAKE BIG POSITIVE PROFITS.

- IF REVENUE ENTERS THE "GOOD REGION", THEN POSITIVE PROFIT IS POSSIBLE!
- IN THE "BAD REGION", PROFIT IS NEGATIVE BUT NOT LOSING AS MUCH AS FIXED COSTS (STAY OPEN)



• IN "UGLY REGION", WILL ALWAYS LOSE MORE MONEY THAN FC (SHUT DOWN!).